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Monopoles and the Emergence of Black Hole Entropy

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Abstract

One of the remarkable features of black holes is that they possess a thermodynamic description, even though they do not appear to be statistical systems. We use self-gravitating magnetic monopole solutions as tools for understanding the emergence of this description as one goes from an ordinary spacetime to one containing a black hole. We describe how causally distinct regions emerge as a monopole solution develops a horizon. We define an entropy that is naturally associated with these regions and that has a clear connection with the Hawking-Bekenstein entropy in the critical black hole limit.

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Black holes have long captured the modern imagination. These objects, containing spacetime singularities hidden behind event horizons, manifest features both striking and surprising. Among these is the fact that thermodynamic properties can be ascribed to black holes, even though they do not appear to be statistical systems. In this essay, we discuss how a thermodynamic description emerges as one goes from a normal spacetime to a spacetime containing a black hole.

How can one investigate the transition from a nonsingular spacetime to one with a horizon? Stars and other astrophysical sources that collapse and form event horizons when they exceed a critical density and size offer one possible direction. However, the onset of black hole behavior happens when the horizon is infinitesimally small, with infinite curvatures; quantum effects that are presumably important here are as yet poorly understood.

Self-gravitating magnetic monopoles [1–5] offer another class of laboratories for investigating the onset of black-hole behavior. They have the advantage of being parametrically tunable systems in which the approach to the black hole limit can be implemented by increasing the soliton mass scale relative to the Planck mass [6]. Furthermore, by appropriate choice of parameters one can make the horizon radius of the critical solution arbitrarily large and the curvatures arbitrarily small, thus ensuring that quantum gravity effects can be safely ignored.

A. Self-gravitating magnetic monopoles

For our purposes it is sufficient to consider spherically symmetric spacetimes, for which the metric can be written in the form

$$ds^2 = Bdt^2 - Adr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \ .$$

In general, a horizon corresponds to a zero of $1/A$; the horizon is extremal if $d(1/A)/dr$ also vanishes. We work in the context of an SU(2) gauge theory with gauge coupling e and a triplet Higgs field whose vacuum expectation value v breaks the symmetry down to U(1). The elementary particle spectrum of the theory includes a neutral massive Higgs particle, a pair of electrically charged vector bosons, and a massless photon.

In flat spacetime this theory possesses a finite energy monopole solution with magnetic charge $Q_M = 4\pi/e$ and mass $M \sim v/e$. This monopole has a core region, of radius $\sim 1/ev$, in which there are nontrivial massive fields. Beyond this core is a Coulomb region in which the massive fields rapidly approach their vacuum values, leaving only the Coulomb magnetic field.

The effects of adding gravitational interactions depend on the value of v. For v much less than the Planck mass $M_{\rm Pl}$, one finds that $1/A$ is equal to unity at the origin, decreases to a minimum at a radius of order $1/ev$, and then increases again with $A(\infty) = 1$. As v is increased, this minimum becomes deeper, until an extremal horizon develops at a critical value $v_{\rm cr}$ of the order $M_{\rm Pl}$; interestingly, the interior remains nonsingular. We will refer to the radius, $r = r_*$, at which $1/A = (1/A)_{\rm min}$ as the quasi-horizon.

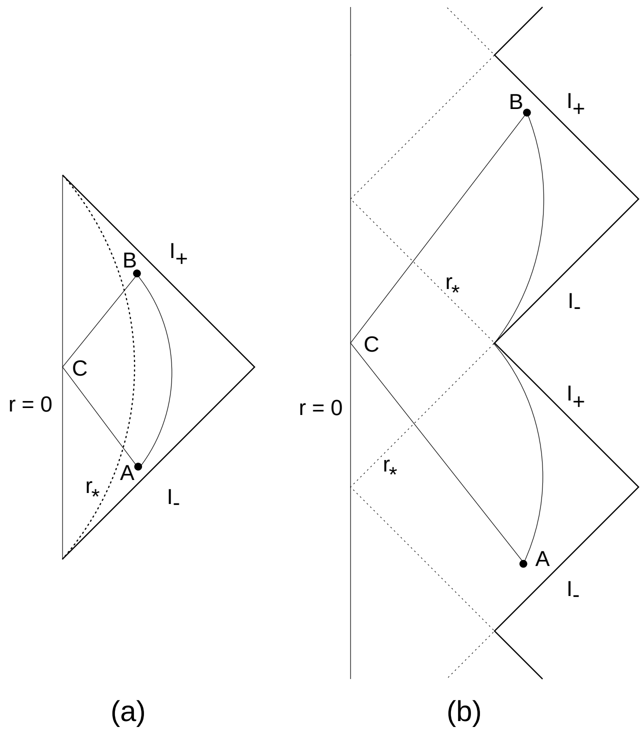

FIG. 1. Penrose diagrams for (a) subcritical monopole and (b) critical monopole black hole. In the former case r_* represents the quasi-horizon whereas in the latter case that radius represents a true horizon.

B. Probing the quasi-black hole

For any $v < v_{\rm cr}$, the self-gravitating monopole solution is a nonsingular spacetime with a Penrose diagram of the same form as that of Minkowski spacetime (Fig. 1a). The critical solution, on the other hand, can be extended beyond the original coordinate patch to yield a spacetime with the Penrose diagram shown in Fig. 1b. This diagram is quite similar to that of an extremal RN black hole, but differs from it by not having a singularity at $r = 0$. The difference between the two diagrams is striking and seems to indicate a discontinuity at $v = v_{\rm cr}$, in contradiction with the usual expectation that physical quantities should vary continuously with the parameters of a theory. However, this discontinuity is perhaps better viewed as an artifact of the conformal transformation that produces the Penrose diagram from an infinite spacetime. This can be seen by considering an observer who remains at a radius $r = r_{\rm obs} \gg r_*$ and probes the interior of the quasi-black hole by sending in a particle along the trajectory ACB shown in Fig. 1a. As the probe moves along this trajectory, the elapsed coordinate time (which is approximately the same as the elapsed proper time of the observer) is

$$\Delta t = 2\int_0^{r_{\rm obs}} dr\, \frac{dt/d\tau}{dr/d\tau} = 2\int_0^{r_{\rm obs}} dr\, \frac{A}{\sqrt{AB}} \left[1 - \frac{B}{E^2}\left(\frac{J^2}{r^2}+1\right)\right]^{-1/2}$$

where E is the probe's energy and J is its angular momentum.

Consistency with our physical expectations of continuity requires that Δt diverge as the quasi-black hole approaches the critical limit and $\epsilon = (1/A)_{\rm min} \to 0$. If this happens, then the region containing point B would become effectively disconnected from that containing point A, just as in the black hole Penrose diagram of Fig. 1b. By examining the behavior of the metric functions as the quasi-black hole approaches the critical limit, we find that in this limit

$$\Delta t \approx k\epsilon^{-q} + \cdots$$

where the exponent q depends on specific monopole parameters but is always greater than or equal to 0.5. A similar result is obtained if one considers probing the black hole interior by sending in waves of some classical field. Thus, the time needed for an external observer to obtain information from the interior region diverges in the critical limit. Most importantly, the leading contribution to Δt is determined solely by the spacetime geometry and is independent of the energy, angular momentum, or other features of the probe.

C. Entropy and thermodynamics

Until a horizon is actually formed, the interior of the quasi-black hole can be probed by external observers of infinite patience and lifetime. However, for an observer with a finite lifetime T, the interior region of a near-critical configuration becomes inaccessible once $\epsilon \lesssim T^{-1/q}$. Such an observer would most naturally describe any larger system containing this configuration in terms of a density matrix ρ obtained by tracing over the degrees of freedom inside the quasi-horizon. Using this density matrix one can define an entropy $S_{\rm interior} = -{\rm Tr}\,\rho \ln \rho$ that can be associated with the interior of the quasi-black hole.

One could, of course, proceed in this manner to define an entropy for any arbitrary region in space, just as one can choose to make the information in any subsystem inaccessible by putting the subsystem behind a locked door. The crucial difference here is that the inaccessibility is due to the intrinsic properties of the spacetime, and that the boundary of the inaccessible region is defined by the system itself rather than by some arbitrary external choice. It is thus reasonable to define $S_{\rm interior}$ as *the* entropy of the quasi-black hole.

A precise calculation of this entropy is clearly infeasible. Among other problems, such a calculation would require a correct implementation of an ultraviolet cutoff, which presumably would require a detailed understanding of how to perform the calculation in the context of a consistent theory of quantum gravity. As an initial effort, one can take the ultraviolet cutoff to be the Planck mass $M_{\rm Pl}$ and try to obtain an order of magnitude estimate. To see what result might be expected, we recall a calculation of Srednicki [7], who showed that tracing over the degrees of freedom of a scalar field inside a region of flat spacetime with

surface area A leads to an entropy $S = \kappa M^2 A$, where M is the ultraviolet cutoff and κ is a numerical constant. Although the value of κ depends on the details of the theory, general arguments [7] suggest that an entropy obtained in this fashion should always be proportional to the surface area. Hence, we expect that the entropy associated with our quasi-black hole is $S_{\rm interior} \sim M_{\rm Pl}^2 A$. A very plausible guess is that in the critical limit this goes precisely to the standard black hole result $S_{\rm BH} = M_{\rm Pl}^2 A/4$.

The suggestion that the entropy of a black hole might be understood in terms of the degrees of freedom inside the horizon is not a new idea. However, any attempt to make this idea more precise must overcome the difficulties that the "interior" region of a black hole is not static and that it contains a singularity. In contrast, our spacetime configurations are static and topologically trivial. Because their interiors can be unambiguously defined, it is conceptually clear what it means to trace over the interior degrees of freedom, even though it may not yet be possible to implement this calculation in complete detail.

Our calculations suggest an understanding of how a thermodynamics description emerges as one moves from a flat space configuration to a black hole. In any thermodynamic description of a system there is an implicit time scale τ that separates fast processes accounted for in the thermodynamics from slow process that are not. One assumes that for times shorter than τ the system can be described as effectively in equilibrium; for true equilibrium, this time scale is infinite. Correspondingly, self-gravitating monopoles have an associated time scale that gives the minimum time needed for an external observer to probe the interior; on shorter time scales, the monopole is effectively a statistical system as far as the observer is concerned. In the limit where the horizon forms and the monopole becomes a true black hole, this time scale becomes truly infinite and the thermodynamic description becomes exact.

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